DETECTION LIMITS IN THE FAR INFRARED

G. H. RIEKE and E. T. YOUNG
Steward Observatory
University of Arizona
Tucson, AZ. 85721

T. N. GAUTIER
Jet Propulsion Laboratory
Pasadena, CA. 91109

Abstract.
The advent of far infrared arrays will change fundamentally the means of analyzing observations in this spectral region. Sources much fainter than traditional "confusion limits" will be extracted from images by using computer algorithms similar to CLEAN or DAOPHOT. We have conducted numerical experiments to evaluate these techniques and show that they will permit long integrations ($\sim$ 10,000 sec at 60μm, $\sim$ 200 sec at 100μm) to achieve nearly photon-background-limited performance and hence very deep detection limits. The dominant noise sources – photon noise, confusion by distant galaxies, and confusion by IR cirrus – scale with nearly the same power of the telescope aperture. As a result, the integration times required to reach confusion limits are nearly aperture-independent.

Key words: confusion limit, infrared arrays

1. Introduction

In planning far infrared astronomy missions, it is important to understand the types of noise they will encounter. Confusion by faint galaxies and by infrared cirrus emission can limit the signal to noise achievable in the far infrared. We calculate the magnitude of these effects where the telescope beam is limited purely by diffraction and the confusing galaxies appear as unresolved point sources. These assumptions are appropriate for the majority of envisioned far infrared telescopes, which are designed with image quality and pointing stability required for near infrared operation and which are therefore virtually perfect optical systems in the far infrared.

A number of previous works have argued that natural-background-limited telescopes will be severely limited by confusion noise in a few seconds of integration. These authors have neglected the improvements in performance that will be possible with imaging arrays in the far infrared. When used with arrays in an optimum manner, a telescope cold enough to be photon oise limited from celestial backgrounds has far superior performance to warmer telescopes throughout the far infrared. Telescopes sufficiently cold to be natural background limited can be essentially photon noise limited at wavelengths of 100μm and shorter for long integrations (e.g., as long as 10,000 sec at 60μm).

2. Point Source Confusion Noise

Condon (1974) demonstrates that estimating point source confusion noise requires only knowledge of the integral source density and of the field of view of the instrument, so long as the point source response is taken to be the "effective beam solid angle", Ω_e, which accounts for the wings of the beam. In the regime of interest to us, the integral source counts go approximately as a power law with index $\beta = -1$. In this case, the limiting noise goes as the square of the beam diameter. This steep dependence can lead to substantial variations in estimates of the confusion noise limit if differing assumptions are made about the achievable beam diameter.

IRAS observations provide a direct measure of the far infrared source density. At 60μm, Hacking and Houck (1987) and Hacking (1994, private communication) find ~ 20 objects deg^{-2} brighter than 50 mJy. Above this flux limit, the integral counts follow a power law as $S^{-3/2}$, as expected for Euclidean space. However, below this flux level the most prevalent galaxies become increasingly more distant, and cosmological corrections will become important so that the counts will deviate from the Euclidean dependence. Therefore, a model of the behavior of galaxies must be constructed from the IRAS data and used with an appropriate cosmology to extrapolate to lower flux densities and determine the density of faint galaxies on the sky. The model we have used will be described elsewhere (Rieke, Young, and Gautier, in preparation); here, we note that the source counts from our model are in good agreement (within a factor of ~ 1.5) with the predictions of Franceschini et al. (1991) and Helou and Beichman (1991). At the flux densities and wavelengths of interest for limiting far infrared measurements with cryogenic observatories, all the counts behave roughly as power laws with $\beta \sim -1$. The predictions are not strongly dependent on cosmological assumptions or on the amount of galaxy evolution when normalized to the 60μm counts described above.

Given the densities of far infrared galaxies on the sky, to determine the confusion limit of an instrument we need to determine the smallest effective beam area that it can use. For future missions, the effective beam area should be determined using source extraction techniques appropriate to observations with imaging arrays such as will soon be available for the far infrared (Young et al. 1993). In the following, we investigate whether optimal source extraction with oversampled array images will allow useful data to be obtained in crowded fields that might be considered confusion limited with a single detector. We have taken a numerical approach that allows us to simulate the operation of various source extraction procedures using Monte Carlo methods. We have verified that our results agree with the analytic treatment of Condon (1974) where the results can be compared. In our Monte Carlo simulations, we have combined the effects of confusion and photon noise so that the results realistically model the effects of amplification of the photon noise as the extraction technique operates in increasingly crowded fields. In comparing results with varying pixel sizes, we have assumed that the pixels operate at the photon noise background limit. We have also simulated problems where the required signal to noise is modest, so that small errors in determining the beam profile do not affect our results. These latter two assumptions allow us to address a "fundamental" confusion limit that will not depend critically on the assumed parameters for the instrument.

The definition of confusion limit depends on the nature of the investigation. For example, an astronomer making an unbiased survey for infrared-bright galaxies would be less annoyed at being "confusion limited" in such objects than would one wishing to determine the flux density of a quasar in the same set of data. In the following, we have considered only the second type of observation since it yields the most stringent measure of the confusion limit.

In our experiments, we generated artificial data by drawing sources randomly from a power law distribution $\mathrm{N}(>S) = \mathrm{C}\ S^{-\beta}$, where S is the flux density and C is a constant. Sources were placed at random positions in a field of 57x57 pixels at an average density of 25 per λ/D diameter beam, where D is the telescope aperture. The faintest sources were therefore roughly only 1% of the flux densities at the achievable detection limits. We left out of this field any sources at more than 1000 times the rms noise level, under the assumption that any deep survey field would be selected to avoid extremely bright sources. Each source was convolved with an Airy pattern of full width at half maximum of 10 pixels and gaussian-distributed random noise was added to each pixel. This noise was scaled with pixel size under the assumption of background limited operation. A test source of known amplitude and with an Airy pattern profile was placed in the center of this "data" array.

Initial tests set $\beta = -1$. We used a source extraction method closely related to the CLEAN algorithm. The data array was deconvolved by identifying the brightest pixel and subtracting an Airy pattern of amplitude 1/3 the rms gaussian noise. The position and amplitude of this subtracted flux contribution were stored in another "deconvolved" array and the procedure was repeated, incrementing the amplitudes in the deconvolved array as small amplitude sources were subtracted from the data array. The subtraction was stopped when the variance in the data array increased with subtraction of an additional small source.

The program then computed the estimated flux density of the test source in the deconvolved array. The sky level was determined as the average of the surface brightness between radii of 1 and 2.5 λ/D, after rejecting all peaks at greater than 3 times the rms noise in this region. This rejection was based on the hypothesis that one would avoid obvious neighboring sources in computing a sky level. The source flux density was determined by integrating the signal within apertures of various sizes and subtracting the sky contribution. 400 Monte Carlo integrations were run for each value of assumed noise. It was found that the optimum extraction procedure in the deconvolved image was to integrate the signal within a sharp-sided angular aperture of diameter 0.5 to 0.7 λ/D. Extractions in this size (and neighboring size) apertures reproduced the input source strengths accurately, with no significant biases toward under- or over-estimation.

The artificial central source was selected so that the final signal to noise (including confusion noise) would be near five. This case is appropriate to a deep survey where source detections are achieved down to the limiting noise. For each Monte Carlo run of 400 cases, we determined the rms fluctuations in the estimated brightness of the central source, so the suite of runs yielded the relation between density of confusing sources on the sky and noise in the measurement of the central source. We fitted this relation to the expected theoretical behavior, with the effective beam diameter as a free parameter; an excellent fit was achieved with an assumed beam of angular diameter 0.8 λ/D. That is, in this application and with the CLEAN method of source extraction, we would successfully predict the total noise of the system including amplification of the photon shot noise if we assumed we were observing with a sharp sided aperture of diameter 0.8 λ/D. The results were found to be largely independent of the selection of reference field, so long as the inner radius was $\geq$ 0.6 λ/D. Our results represent a modest amount of superresolution over the full width at half maximum of 1.03 λ/D for an Airy function.

A second set of simulations was run with a power law source index of $\beta = -1.5$, with results very similar to those just described. Again, the behavior is adequately described by the simple theory if a beam diameter of 0.8 λ/D is assumed. The experiments described above assumed an extreme degree of oversampling, i.e., pixels of 0.1 the diffraction-limited beam diameter. An additional set of experiments addressed the use of larger pixels. Here, the data frames and CLEAN beam were both measured with pixels respectively of 0.3, 0.5, and 0.7 λ/D. It was assumed that the telescope was substepped to maintain the same sampling interval as before, that is 0.1 λ/D. For example, with the 0.5 λ/D pixels, it would be required that data be taken at 25 telescope pointings on a 5x5 grid with 0.1 λ/D between points. 200 iterations were made for each case, all at the same confusing source density where the previous experiments indicated confusion noise would degrade the photon noise limit by a factor of 3. These experiments produced similar results to those with the 0.1 λ/D pixels, showing that imagers can make efficient use of the far infrared arrays by using pixels that are a reasonably large portion of the Airy pattern. ¿From other experiments, we conclude that this favorable result arises only if the telescope is substepped on a finer grid than the pixel-to-pixel spacing. In addition, the requirements on the accuracy of calibration and the knowledge of the point spread function will be increased as the pixel size is increased. These factors must all be considered in the design of realistic systems for high sensitivity operation in the far infrared.

Although we have treated our simulations as if there is a hard confusion limit, at higher ratios of signal to noise it is likely that the source could be localized more precisely than in our experiments, leading to a smaller effective beam diameter and an improvement in limiting flux density. Again, achieving these goals will place greater demands on the calibration and knowledge of the point spread function and therefore will depend on the details of system design.

As examples, we compute for an 85 cm telescope at 60, 100, and 150μm the rms confusion noise limits due to distant galaxies given in Table I. The confusion noise from point sources with uncorrelated positions on the sky will scale in this case inversely as the effective aperture area, or as D^{-2}.

Table I. Noise Components for 1000 Sec Integrations

Noise Source	60μm	100μm	150μm
Galaxies	$\sim 25\mu$Jy	350μJy	1500μJy
IR Cirrus	6μJy	89μJy	300μJy
Photon Noise	90μJy	140μJy	400μJy
Net Noise	94μJy	387μJy	1580μJy

3. Confusion by Infrared Cirrus

We base our discussion of confusion by infrared cirrus on the formalism of Gautier et al. (1992). To apply these results, we need to set three parameters: 1.) the power law index, α, of the cirrus emission power; 2.) the power at 0.01 arcmin^{-1}, P_o(0.01/arcmin); and 3.) the observing geometry.

Analysis of the IRAS far infrared data shows that the cirrus power spectrum has a spectral index of $\alpha = -2.6$ to -3.2 (Gautier et al. 1992, T. N. Gautier unpublished work). The IRAS data do not extend to the high spatial frequencies that are important for future missions that will have both telescopes of larger aperture and imaging far infrared arrays. However, the power spectrum can be estimated at these high frequencies by comparison with deep CCD exposures in the visible, which can reach 1 arcsec^{-1}. Cutri (private communication) finds that the power spectrum steepens slightly from the low spatial frequencies (i.e., becomes more negative). In the following, we will assume $\alpha = -3.0$.

The level of cirrus contamination is characterized by P_o. As a medium level of cirrus contamination, we adopt the average P_o for $|\mathrm{b}| > 50^o$, namely 3×10^6 Jy^2/sr (Gautier et al. 1992). Other levels of cirrus emission are listed in Table II.

Table II. Cirrus Confusion Noise at 100μm

Type of Area	P_o(0.01 arcmin^{-1}) (Jy^2/sr)	RMS Confusion (μJy/beam)
Best 100 deg^2 in Baade's Hole	$\leq 1.2 \times 10^4$	≤ 6
500 additional deg^2 in Baade's Hole	$\leq 5 \times 10^5$	≤ 37
Average, $|\mathrm{b}| \geq 50^o$	3×10^6	89
Moderately Bright Cloud	10^8	510
Bright Cloud	10^{10}	5100

Because of the steep power spectrum of the cirrus emission, there is a premium in keeping the reference field as tightly coupled to the source as possible. We have therefore computed the cirrus confusion noise assuming an annular reference field that lies between 0.6 and 1.6 λ/D. For comparison with the galaxy confusion limit, we have interpolated to an effective beam diameter of 0.8 λ/D. Because of the low power at high spatial frequencies, the noise from cirrus confusion is only weakly dependent on the beam diameter but very strongly on the reference area size.

For the case of an 85 cm telescope, the resulting cirrus confusion noise limit at the average level of high latitude cirrus emission is 89μJy per beam, with these assumptions. Values for the rms confusion noise for the 85 cm telescope at other wavelengths and in other parts of the sky are entered in Table II. The confusion noise will scale as the square root of P_o and as $D^{(\alpha/2-1)}$, i.e., as $D^{-2.5}$ for $\alpha = -3$.

4. Photon Noise

A far-infrared astronomical system will also be limited by photon noise. The cosmic background at infrared wavelengths can be broken down into a number of important components. At shorter wavelengths scattered solar radiation and thermal emission from zodiacal dust dominate, while beyond $\sim$ 40μm emission from cold galactic dust must be included. Each of these components has a distinctive positional dependence on the sky, and large variations in the relative contributions are present. To investigate a case appropriate for extragalactic astronomy, we have taken the COBE Diffuse Infrared Background Experiment (DIRBE) measurements of the South Galactic Pole (Hauser et al. 1991). The noise from this background is computed in a conventional manner (e.g., Rieke 1994); because the astronomical backgrounds are due to very dilute graybodies, the Boson correction can be ignored (van Vliet 1967).

Since the focal plane fully samples the point spread function, the signal is spread over a number of pixels. Therefore, we combine the measurements from the number of pixels needed to synthesize a point source, quadratically adding the noise. An additional noise penalty of a factor of 1.1 was added to allow for noise in the flat field. We have assumed a diffraction limited beam (for a telescope with 15% areal obscuration) and have computed the net signal in apertures centered on the source after subtraction of the average surface brightness in a reference area between 1 and 2.5 λ/D. The result is that the signal to noise for a point source is 8.2 times worse than that computed for a single 0.4 λ/D pixel (assuming all the signal fell on the single pixel) if the signal is measured in a 0.8 λ/D beam and 7.1 times worse if measured in a 1.2 λ/D beam. The latter beam size has the maximum ratio of signal to noise if photon noise alone is considered; however, to maximize the ratio of signal to noise that can be achieved in a confusion-limited situation, we will use the former beam size in the following.

Some degradation of the photon noise will occur in space because cosmic rays striking the detector will limit the integration times. A discussion of the extent of this effect is given by Herter (1990). The results depend on detector geometry, read noise, and the method of operation of the readout. We assume appropriate parameters for the far infrared arrays under development for SIRTF (Young et al. 1993), i.e., that a cosmic ray hit destroys all information in the integration after the hit, a hit affects the pixels on either side of the hit one so that the resulting net pixel area is 0.13 cm^2, and the read noise is 50 electrons; the result is that the pure background-limited photon noise will be degraded by a factor of ~ 1.5.

The photon noise limits in Table I are computed according to the description above and include the degradation by cosmic radiation. Of course, for a diffraction limited telescope, these detection limits scale as D^{-2}.

5. Combined Noise

The various sources of noise are combined in Table I, where the cirrus emission has been taken at the average for high latitude sky. The behavior across the far infrared band progresses rapidly toward increasing levels of confusion with increasing wavelength, both because of the increase in the density of galaxies of a given flux density and because of the increase in the diffraction limited beam size. At 60μm, integrations greater than 10,000 seconds are required to approach serious levels of confusion, while 200 seconds and 100 seconds are required respectively at 100 and 150μm. It is possible with an 85 cm telescope to reach limits about two orders of magnitude deeper than the data in IRAS bands 3 and 4.

We note that the scaling of the noise sources, for a given and roughly optimized method of source extraction, is D^{-2} for point source confusion noise and photon noise and $D^{-2.5}$ for cirrus noise. Thus, the relative importance of these differing noise sources will change only slowly with telescope aperture. Any telescope designed to operate at fundamental detection limits in the far infrared should be cold enough so that the photon noise does not increase significantly compared with the natural background.

6. Conclusion

The advent of far infrared detector arrays will allow qualitatively new approaches to sensitive astronomical detection at these wavelengths. By applying source extraction techniques similar to CLEAN or DAOPHOT, the effective beam size can be reduced substantially with a resulting reduction in confusion noise compared with predictions for instruments with a small number of detectors or for aperture photometers. The same benefits can be achieved for operating in regions where there is strong infrared cirrus. To operate in this mode, it is essential that far infrared telescopes have highly reproducible, preferably diffraction limited, beam profiles.

Table I shows as an example the sensitivity that can be achieved with a cryogenic 85 cm telescope using a CLEAN algorithm and operating at the average level of IR cirrus at high Galactic latitude. At 60μm, the instrument will not become confusion limited unless integrations longer than 10,000 sec are attempted.

Because the three types of limiting noise scale with almost the same power of the telescope aperture, their relative contributions will not change significantly for telescopes of other sizes. As a result, the well-recognized benefits of operating meter-class telescopes at temperatures cold enough that they are natural background limited also apply to larger instruments. For example, although a telescope operating at $<$ 5K at 160 - 200μm will become confusion limited in about 60 seconds, the same telescope at 8K would require 1000 seconds to reach the same sensitivity limit; at 15K it would require 100,000 seconds. Therefore, designs for future far infrared telescopes should emphasize operating them at low temperatures.

Acknowledgements

We greatly appreciate the hospitality of the conference organizers. The work reported here was supported by NASA as part of the SIRTF project.

References

Condon, J. J.: 1974, *ApJ* **188**, 279

Franceschini, A., Toffolatti, L., Mazzei, P., Danese, L., and De Zotti, G.: 1991, *A & A Suppl.* **89**, 285

Gautier, T. N., Boulanger, F., Perault, M., and Puget, J. L.: 1992, *AJ* **103**, 1313

Hacking, P., and Houck, J. R.: 1987, *ApJS* **63**, 311

Hauser, M. G., Kelsall, T., Moseley, S. H., Silverberg, R. F., Murdock. T., Toller, G., Spiesman, W., and Weiland, J.: 1991, 'The First 3 Minutes' in S. S. Holt, C. L. Bennett, and V. Trimble, ed(s)., *AIP Conf. Proc. 222*, Am. Inst. of Phys.:New York, 161

Helou, G., and Beichman, C. A.: 1991, *Proc. 29th Liege International Astrophysical Colloquium* **ESA SP-314**, 117

Herter, T.: 1990, *IR Phys.* **30**, 97

Rieke, G. H.: 1994, *Detection of Light*, Cambridge University Press

van Vliet, K. M.: 1967, *App. Opt.* **6**, 1145

Young, E. T., Scutero, M. J., Rieke, G. H., Haller, E. E., and Beeman, J. W.: 1993, *SPIE* **1946**, 68